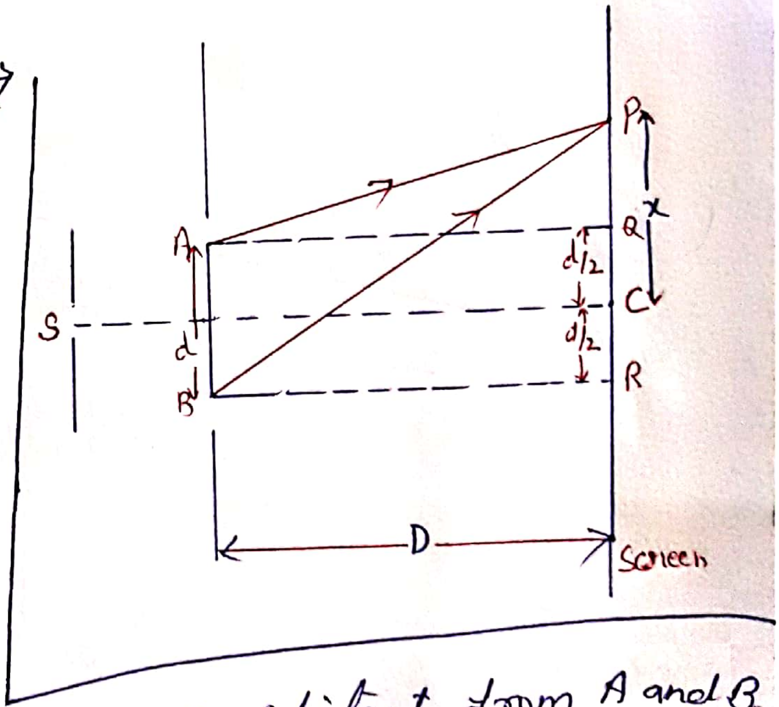
Unit – III Interference

Interference of light → light is form of electromagnetic wave which is transverse in nature. A single source of light gives out energy which is distributed uniformly in the surrounding medium. If two independent sources of light giving out continuous waves of same freq- uency, same amplitude, same wavelength and same phase or constant phase difference are held close to each other, the distribution of light is not uniform, but bright and dark regions are observed.

At places where the crest of one wave falls on the crest of the other wave or a trough of one falls on the trough of the other, the amplitude increases and hence the energy or intensity of light is maximum and it is called constructive interference.

At places where trough of one falls on the crest of the other, the resultant amplitude is zero and hence intensity is minimum and it is destructive interference. This gives rise to a phenomenon of interference.

Theory of Interference fringes →



Consider a monochromatic source of light S emitting waves of wavelength λ and pin holes A and B. A and B are equi- distant from S, acts as two coherent sources separated by a distance 'd'.

Let a screen be placed at a distance 'D' from the coherent sources. The point C on the screen is equidistant from A and B. Therefore, path difference between two waves is zero. Thus the pt. C has maximum intensity

Consider a point P at a distance 'x' from C. The waves reach at the point P from A and B.

Here $PQ = x - \frac{d}{2}$, $PR = x + \frac{d}{2}$

∴ Path difference between two waves reaching at P = BP − AP

In rt. ∠△BRP, we have, $(BP)^2 = (BR)^2 + (PR)^2$

$$= D^2 + (x + d/2)^2 \quad \text{— (1)}$$

In rt. ∠△AQP, we have $(AP)^2 = (AQ)^2 + (PQ)^2$

$$= D^2 + (x - d/2)^2 \quad \text{— (2)}$$

Subtracting =n (2) from =n (1), we get

$$(BP)^2 - (AP)^2 = \cancel{D^2} + \left(x + \frac{d}{2}\right)^2 - \cancel{D^2} - \left(x - \frac{d}{2}\right)^2$$

$$(BP + AP)(BP - AP) = \cancel{x^2} + \cancel{\frac{d^2}{4}} + \frac{2xd}{2} - \cancel{x^2} - \cancel{\frac{d^2}{4}} + \frac{2xd}{2}$$

$$= 2xd$$

$$\therefore \quad BP - AP = \frac{2xd}{BP + AP}$$

But BP = AP = D (approx.) as P lies close to A and B

$$\therefore \quad BP - AP = \frac{2xd}{D + D} = \frac{2xd}{2D} = \frac{xd}{D}$$

$$\therefore \quad \boxed{\text{Path difference} = BP - AP = \frac{xd}{D}} \quad \text{— (3)}$$

<u>Bright Fringes (Constructive Interference)</u> → If path difference is integral multiple of 'λ' (wavelength), the point P is bright. Thus for bright fringes,

$$\text{Path difference} \quad \frac{xd}{D} = n\lambda$$

$$x = \frac{n\lambda D}{d} \quad \text{— (4)}$$

Thus for $n = 0$, $x_0 = 0$ (Central bright fringe)

for $n = 1$, $x_1 = \frac{\lambda D}{d}$ (first bright fringe)

for $n = 2$, $x_2 = \frac{2\lambda D}{d}$ (second bright fringe) and so on

$\therefore$ Distance between two consecutive bright fringes

$$= x_2 - x_1 = \frac{2\lambda D}{d} - \frac{\lambda D}{d} = \frac{\lambda D}{d} \quad — (5)$$

Dark fringes (Destructive Interference) → If the path difference is an odd multiple of $\lambda/2$, the point P is dark.

Thus for dark fringes, Path difference $\frac{xd}{D} = (2n+1)\frac{\lambda}{2}$

$$\therefore x = \frac{(2n+1)\lambda D}{2d} \quad — (6)$$

Thus for $n=0$, $x_0 = \frac{\lambda D}{2d}$ (first dark fringe)

for $n=1$, $x_1 = \frac{3\lambda D}{2d}$ (Second dark fringe)

for $n=2$, $x_2 = \frac{5\lambda D}{2d}$ (Third dark fringe) and so on

Distance between two consecutive dark fringes

$$= x_2 - x_1 = \frac{5\lambda D}{2d} - \frac{3\lambda D}{2d} = \frac{\lambda D}{d} \quad — (7)$$

The distance between any two consecutive bright or dark fringes is known as fringe width. Both the bright and dark fringes are of equal width.

Thus, <u>Fringe width</u> $\beta = \frac{\lambda D}{d} \quad — (8)$

Thus width of a fringe increases

(a) with Increase in the wavelength 'λ'

(b) with Increase in the distance 'D'

(c) by bringing the two sources A and B close to each other.

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Interference in thin films →

It is due to the effect of interference that brilliant colours are seen, when a thin layer of oil is spread on water or thin film of soap solution is viewed in broad day light.

Interference in case of thin films takes place due to

1. Reflected light and
2. Transmitted light.

1. Interference due to reflected light →

Consider a thin film of thickness 't' and refractive index 'μ' bounded by two plane surfaces PQ and RS. A ray of light coming from monochromatic source be incident on the surface PQ at an angle 'i'. It is partly reflected along BC and partly refracted along BD at an angle of refraction 'r'. At 'D' a part of it is reflected along DE and finally emerges out along EF as shown in fig ①. These two rays along paths BC and BDEF are derived from same incident ray (AB) are coherent and can produce interference.

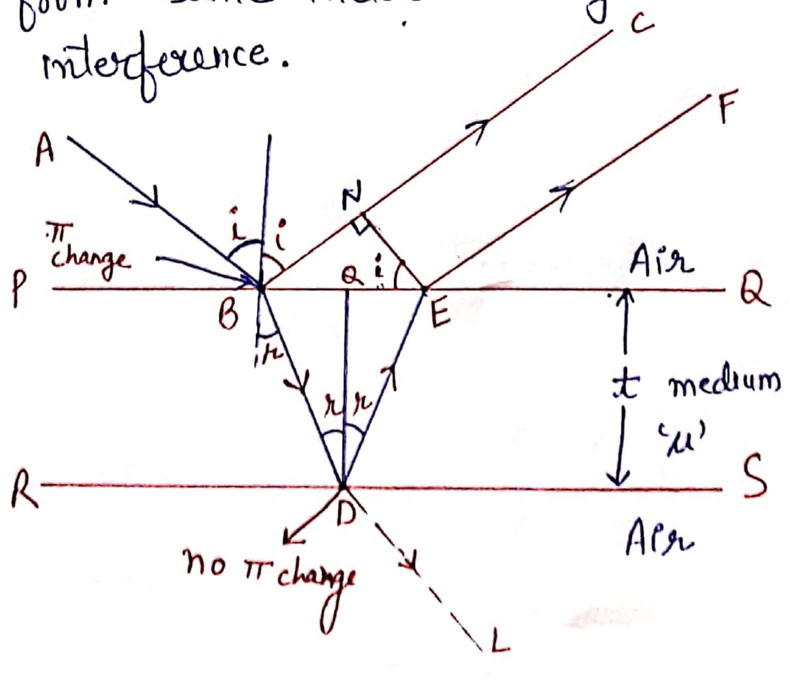


fig ①

In ΔBDE

$\angle BDQ = \angle EDQ = r$

$BD = DE$

$BQ = QE$

$90 + 90 - i + E = 180$

$\angle E = i$

(i) Geometrical Path difference:

To find the path difference between two rays BC and EF, draw $EN \perp BC$. The ray 'BN' travels in air while the ray BE travels in the film along path BD and DE.

$\therefore$ Geometric path difference $= BD + DE - BN$

(ii) Optical Path difference:

Optical Path difference $= \mu \times$ Geometrical path difference

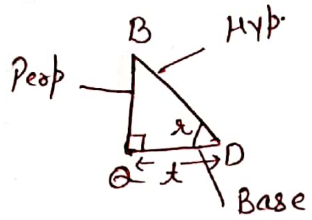
$$= \mu\{(BD+DE) - BN\}$$

$$= \mu(BD+DE) - \mu(BN)$$

$$= \mu(BD+DE) - BN \quad \because \text{ for air } \mu = 1$$

$$\text{—— (1)}$$

Now In rt $\angle\triangle$ BDQ

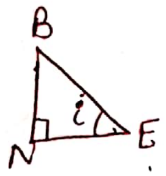


$$\frac{BD}{DQ} = \frac{1}{\cos r}$$

$$BD = \frac{DQ}{\cos r} = \frac{t}{\cos r}$$

$$\therefore BD = DE = \frac{t}{\cos r} \quad \text{—— (2)}$$

Also $\frac{BQ}{DQ} = \frac{\text{Perp.}}{\text{Base}} = \tan r \Rightarrow BQ = DQ \tan r = t \tan r$ —— (3)



In rt$\angle\triangle$ BNE, $\angle NBE = 90 - i°$,
$\angle BNE = 90°$
$\therefore \angle BEN = i$

$\frac{BN}{BE} = \sin i \Rightarrow BN = BE \sin i = (BQ + QE)\sin i = 2t \tan r \sin i$ —— (4) $\because$ of (3)

Using eqn. (2), 3, (4) in eqn. (1),

$$\text{Path difference} = \mu\left(\frac{t}{\cos r} + \frac{t}{\cos r}\right) - 2t \tan r \sin i$$

$$= \frac{2\mu t}{\cos r} - 2t\frac{\sin r}{\cos r}\sin i,$$

From Snell's law, $\mu = \frac{\sin i}{\sin r} \Rightarrow \sin i = \mu \sin r$

Path difference $= \frac{2\mu t}{\cos r} - \frac{2t \sin r}{\cos r} \mu \sin r$

$= \frac{2\mu t}{\cos r}(1 - \sin^2 r) = \frac{2\mu t}{\cos r} \cos^2 r$

$$\boxed{\text{Path difference} = 2\mu t \cos r}$$

(iii) Correction on account of phase change at reflection:

Since reflection is taking place at the surface of denser medium, so an additional phase change of π or additional path difference of $\lambda/2$ is introduced.

$\therefore$ Corrected or net path difference, $x = 2\mu t \cos r - \lambda/2$ — (1)

Condition for Maxima / Brightness / Constructive Interference:

If path difference $x = n\lambda$, Constructive inference takes place and film appears bright.

$\therefore 2\mu t \cos r - \lambda/2 = n\lambda$

or $\boxed{2\mu t \cos r = (2n+1)\lambda/2}$ — (2), $n = 0, 1, 2, 3, \ldots$

Condition for Minima / Darkness / Distructive Interference:

If path difference $x = (2n+1)\lambda/2$ {odd multiple of half wavelength}

distructive interference takes place and film appears dark

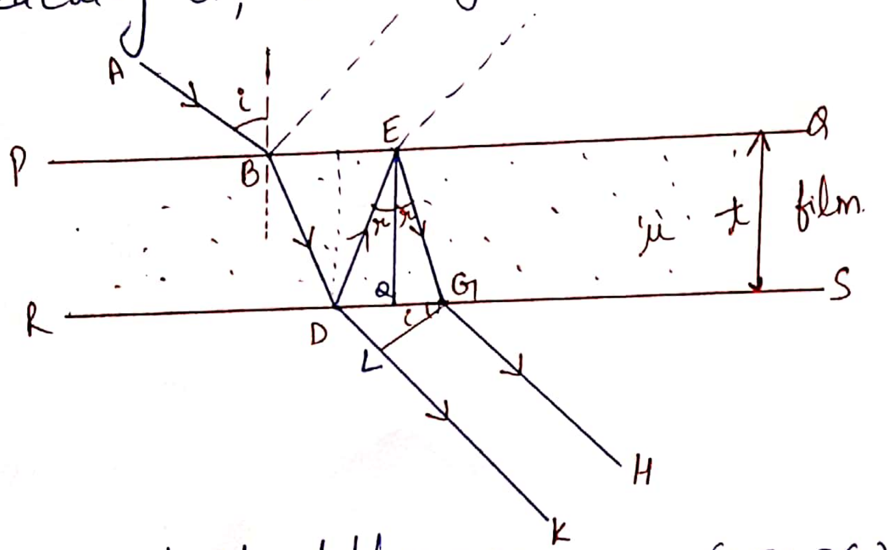
$\therefore 2\mu t \cos r - \lambda/2 = (2n+1)\lambda/2$

$\boxed{2\mu t \cos r = (n+1)\lambda = n\lambda}$ (for simplicity) — (3)

Note: Air-medium Interface (boundary), path difference is introduced
Medium-Air Interface — no additional path diff. is introduced.

II. Interference due to transmitted light :→

Consider a thin transparent film of thickness 't' and refractive index 'μ' bounded by two parallel surfaces PQ and RS. Let a ray of light AB incident on surface PQ is partly reflected along BC and partly refracted along BD. At D, a part of the ray is reflected along DE and another part is transmitted out along DK. From E, it is again reflected along EG and on reaching G, the ray is refracted along GH.



∴ Optical path difference $= \mu(DE + EG) - DL$

Now $DE = EG = \dfrac{t}{\cos r}$ and $DQ = QG = t\tan r$

Also $DL = DG\sin i = (DQ + QG)\sin i = 2t\tan r \sin i$

∴ Path difference $= \dfrac{2\mu t}{\cos r} - 2t\tan r \sin i$

$= \dfrac{2\mu t}{\cos r} - 2t\dfrac{\sin r\, \mu \sin r}{\cos r}$ ∵ $\sin i = \mu \sin r$

$= \dfrac{2\mu t}{\cos r}(1 - \sin^2 r)$

$= \dfrac{2\mu t}{\cos r}\cos^2 r = 2\mu t \cos r$

$$\boxed{\text{Path difference} = 2\mu t\cos r}$$

In this Case there is no phase change on reflection as it takes place at the surface of rarer-medium.

Condition for maxima (Bright fringes)

when path difference $x = n\lambda$, film appears bright

i.e $\boxed{2\mu t \cos r = n\lambda}$ — (4) $n = 0, 1, 2, \ldots$

Condition for minima (Darkness or Dark fringes)

when path difference $x = (2n+1)\lambda/2$, film appears dark

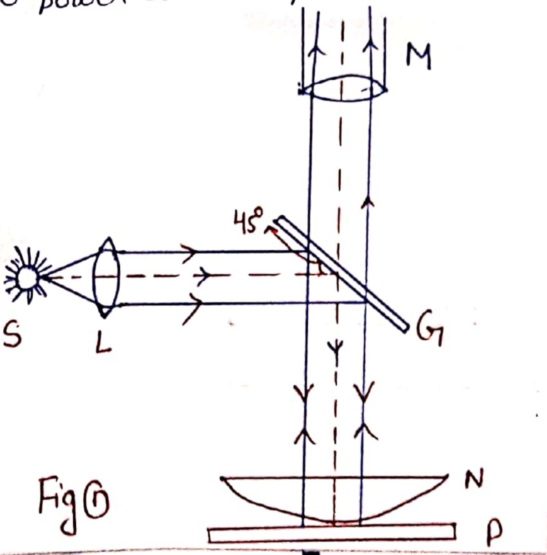
$\therefore$ $\boxed{2\mu t \cos r = (2n+1)\lambda/2}$ — (5)

From eqns (2), (3), (4) and (5), it is clear that Condition of interference for reflected and transmitted system are Complementary. Hence part of film that appear dark in the reflected system will appear bright in the transmitted system and vice versa.

Newtons Rings → Circular interference fringes can be produced by enclosing a very thin film of air or any transparent medium of varrying thickness between a plane glass plate and plano convex lens of large radius of curvature. Such fringes were first obtained by Newton and are known as Newton's rings, These are obtained by reflected as well as transmitted light.

Experimental Arrangement → To produce Newton's concentric rings, monochromatic light from an extended source 'S' is rendered parallel by a convex lens 'L'. It falls on a glass plate 'G' inclined at an angle of 45° to the incident beam and is reflected normally on the surface of plano-convex lens of large radius of curvature placed on a glass plate 'P' as shown in fig①.

Light rays reflected upward from the top and bottom surfaces of the air film formed between the lens and the glass plate 'P' superimpose eachother with a path difference depending upon the air film thickness in between. Due to interference of these rays, dark and bright circular fringes are seen. The fringes are circular because the air film is symmetrical about the pt of contact of lens N with glass plate P. These fringes can be viewed/observed by low power travelling microscope M.



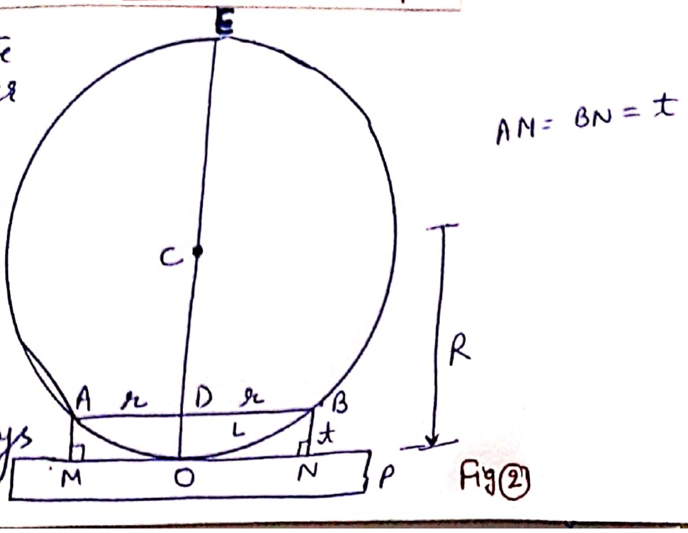
Fig①

Theory:

a) Newton's rings by reflected light →

Let AOB be the vertical section of the lens having radius of curvature 'R'. The lens is in contact with glass plate 'P' at pt O, in such a manner that A and B are equidistant from O. Complete the circle AOBE and draw diameter ODE. Draw BN and AM ⊥ to the plane MN. The thickness of the air film will be zero at 'O'. Around O, circular fringes are produced.

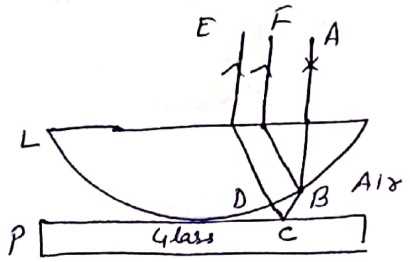
Path difference between two rays



Fig②

one reflected from B (BF) and other reflected from C (CDE) is $2\mu t\cos r$.
Since angle of refraction small $\therefore \cos r \simeq 1$ and for air $\mu = 1$

$\therefore$ Path difference $= 2t$



For Bright rings,

Path difference $= (2n+1)\lambda/2$

i.e $2t = (2n+1)\lambda/2$ — (1)

For Dark fringes (rings),

Path difference $= n\lambda$

i.e $2t = n\lambda$ — (2)

In fig (3), From geometry of the Circle, we have

$$AD \times DB = OD \times DE$$
$$= OD\,(OE - OD)$$
$$r \times r = t\,(2R - t)$$

As t is small as Compared to $2R$, $(2R - t) \simeq 2R$.

$$\therefore r^2 = 2Rt$$

$$2t = \frac{r^2}{R} \quad \text{— (3)}$$

$\therefore$ From (1), for bright rings

$$r^2 = (2n+1)\frac{\lambda R}{2}$$

$$r = \sqrt{\frac{(2n+1)\lambda R}{2}}$$

$\therefore$ Diameter of bright ring 'd' $= 2r =$

$$d = 2r = \sqrt{2(2n+1)\lambda R}$$

From (2), for dark Rings,

$$r^2 = n\lambda R \Rightarrow r = \sqrt{n\lambda R}$$

Diameter of dark ring

$$'d' = 2r = 2\sqrt{n\lambda R}$$

when $n = 0$, radius of dark ring $= 0$

Thus Centre of Newton's rings is dark.

(ii) Newton's rings by transmitted light →

In Case of transmitted light, the interference fringes are produced such that for bright rings, $2\mu t\cos r = n\lambda$

For dark rings, $2\mu t\cos r = (2n+1)\frac{\lambda}{2}$

Since for small r, $\cos r \simeq 1$ and for air $\mu = 1$

$\therefore$ For bright rings, $2t = n\lambda$

For dark rings, $2t = (2n+1)\frac{\lambda}{2}$

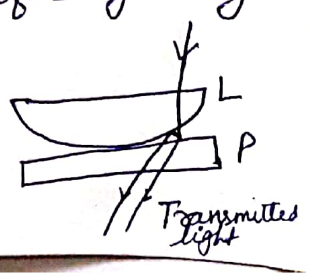
Taking value of $t = \frac{r^2}{2R}$, we have

For bright rings, $r^2 = n\lambda R$

For dark rings, $r^2 = (2n+1)\frac{\lambda R}{2}$

when $n = 0$, radius of bright ring $= 0$

Thus centre of Newton's rings in this Case is bright.



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Applications of Newton's Rings →

1) Determination of wavelength of Sodium light
2) Determination of Refractive index of liquid.

1. Determination of Wavelength of Sodium light →

When a plano convex lens ~~of~~ is placed on a glass plate, then a thin film of air of varying thickness is enclosed between the lens and the plate. When monochromatic light (sodium light) is allowed to fall on this combination, then circular interference fringes (bright and dark) are produced which are viewed through low power microscope.

Let D_n and D_m be the diameter of n^{th} and m^{th} dark ring, then

$$D_n^2 = 4n\lambda R \quad \text{—} \; (i)$$

$$D_m^2 = 4m\lambda R \quad \text{—} \; (ii)$$

Subtracting =n (ii) from (i)

$$D_n^2 - D_m^2 = 4(n-m)\lambda R \qquad n>m$$

$$\lambda = \frac{D_n^2 - D_m^2}{4R(n-m)}$$

knowing the values of D_n, D_m, n, m and R, λ can be determined. R is radius of curvature and can be measured using spherometer. The same results can be obtained by measuring the diameter of n^{th} and m^{th} bright rings.

(2) Determination of Refractive index of liquid →

To determine the refractive index of the liquid, the space between the plate and the convex lens is filled with the given liquid say by placing the plate and the lens in a trough containing the liquid. The diameters of nth and mth ring are determined with the help of travelling microscope

Then for nth dark ring, $D_n^2 = \frac{4n\lambda R}{\mu}$

For mth dark ring, $D_m^2 = \frac{4m\lambda R}{\mu}$

$\because \quad D_n^2 - D_m^2 = \frac{4(n-m)\lambda R}{\mu}$

$$\Rightarrow \boxed{\mu = \frac{4(n-m)\lambda R}{D_n^2 - D_m^2}}$$

Thus knowing n, m, D_n, D_m, λ, R, the refractive index 'μ' can be determined

If wavelength of light (λ) is not known, then determine the diameter of a particular ring first with air film and then with the liquid in the intervening space.

Let D_a and D_l be the diameter of nth dark ring in the two cases, then

$(D_{air})^2 = D_a^2 = 4n\lambda R$

$(D_{liq.})^2 = D_l^2 = \frac{4n\lambda R}{\mu}$

$$\therefore \quad \mu = \left(\frac{D_a}{D_l}\right)^2$$

<u>Newton's Rings with White light</u> → The diameter of a ring depends upon the wavelength of light used and is proportional to the square root of the wavelength [ie $d_n^2 \propto \lambda$ or $d_n = \sqrt{\lambda}$]. If white light is used instead of monochromatic light, coloured rings of different diameters will be produced as white light consists of different colours having different wavelengths. These rings will super-impose and coloured rings will be produced. Only first few rings are clear and after that due to more overlapping, the rings will not be viewed. The central spot even in this case will be dark.